Chapter 6: Uniform Circular Motion and Gravitation

# 6.1 Rotation Angle and Angular Velocity

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| 1. | *Semi-trailer trucks have an odometer on one hub of a trailer wheel. The hub is weighted so that it does not rotate, but it contains gears to count the number of wheel revolutions—it then calculates the distance traveled. If the wheel has a 1.15 m diameter and goes through 200,000 rotations, how many kilometers should the odometer read?* |
| Solution |  |
| 2. | *Microwave ovens rotate at a rate of about 6 rev/min. What is this in revolutions per second? What is the angular velocity in radians per second?* |
| Solution |  |
| 3. | *An automobile with 0.260 m radius tires travels 80,000 km before wearing them out. How many revolutions do the tires make, neglecting any backing up and any change in radius due to wear?* |
| Solution |  |
| 4. | *(a) What is the period of rotation of Earth in seconds? (b) What is the angular velocity of Earth? (c) Given that Earth has a radius of  at its equator, what is the linear velocity at Earth’s surface?* |
| Solution | (a)  (b)  (c) |
| 5. | *A baseball pitcher brings his arm forward during a pitch, rotating the forearm about the elbow. If the velocity of the ball in the pitcher’s hand is 35.0 m/s and the ball is 0.300 m from the elbow joint, what is the angular velocity of the forearm?* |
| Solution |  |
| 6. | *In lacrosse, a ball is thrown from a net on the end of a stick by rotating the stick and forearm about the elbow. If the angular velocity of the ball about the elbow joint is 30.0 rad/s and the ball is 1.30 m from the elbow joint, what is the velocity of the ball?* |
| Solution |  |
| 7. | A truck with 0.4*20-m-radius tires travels at 32.0 m/s. What is the angular velocity of the rotating tires in radians per second? What is this in rev/min?* |
| Solution |  |
| 8. | ***Integrated Concepts*** *When kicking a football, the kicker rotates his leg about the hip joint. (a) If the velocity of the tip of the kicker’s shoe is 35.0 m/s and the hip joint is 1.05 m from the tip of the shoe, what is the shoe tip’s angular velocity? (b) The shoe is in contact with the initially stationary 0.500 kg football for 20.0 ms. What average force is exerted on the football to give it a velocity of 20.0 m/s? (c) Find the maximum range of the football, neglecting air resistance.* |
| Solution | (a)  (b)  (c) |

# 6.2 Centripetal Acceleration

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| 10. | *A fairground ride spins its occupants inside a flying saucer-shaped container. If the horizontal circular path the riders follow has an 8.00 m radius, at how many revolutions per minute will the riders be subjected to a centripetal acceleration whose magnitude is 1.50 times that due to gravity?* |
| Solution |  |
| 11. | *A runner taking part in the 200 m dash must run around the end of a track that has a circular arc with a radius of curvature of 30 m. If he completes the 200 m dash in 23.2 s and runs at constant speed throughout the race, what is the magnitude of his centripetal acceleration as he runs the curved portion of the track?* |
| Solution |  |
| 12. | *Taking the age of Earth to be about  years and assuming its orbital radius of  has not changed and is circular, calculate the approximate total distance Earth has traveled since its birth (in a frame of reference stationary with respect to the Sun).* |
| Solution |  |
| 13. | *The propeller of a World War II fighter plane is 2.30 m in diameter. (a) What is its angular velocity in radians per second if it spins at 1200 rev/min? (b) What is the linear speed of its tip at this angular velocity if the plane is stationary on the tarmac? (c) What is the centripetal acceleration of the propeller tip under these conditions? Calculate it in meters per second squared and convert to multiples of .* |
| Solution | (a)  (b)  (c) |
| 14. | *An ordinary workshop grindstone has a radius of 7.50 cm and rotates at 6500 rev/min. (a) Calculate the magnitude of the centripetal acceleration at its edge in meters per second squared and convert it to multiples of . (b) What is the linear speed of a point on its edge?* |
| Solution | (a)  (b) |
| 15. | *Helicopter blades withstand tremendous stresses. In addition to supporting the weight of a helicopter, they are spun at rapid rates and experience large centripetal accelerations, especially at the tip. (a) Calculate the magnitude of the centripetal acceleration at the tip of a 4.00 m long helicopter blade that rotates at 300 rev/min. (b) Compare the linear speed of the tip with the speed of sound (taken to be 340 m/s).* |
| Solution | (a)  (b) |
| 16. | *Olympic ice skaters are able to spin at about 5 rev/s. (a) What is their angular velocity in radians per second? (b) What is the centripetal acceleration of the skater’s nose if it is 0.120 m from the axis of rotation? (c) An exceptional skater named Dick Button was able to spin much faster in the 1950s than anyone since—at about 9 rev/s. What was the centripetal acceleration of the tip of his nose, assuming it is at 0.120 m radius? (d) Comment on the magnitudes of the accelerations found. It is reputed that Button ruptured small blood vessels during his spins.* |
| Solution | (a)  (b)  (c)  (d) The centripetal acceleration felt by Olympic skaters is 12 times larger than the acceleration due to gravity. That is quite a lot of acceleration in itself. The centripetal acceleration felt by Button’s nose was 39.2 times larger than the acceleration due to gravity! It is no wonder that he ruptured small blood vessels in his spins. |
| 17. | *What percentage of the acceleration at Earth’s surface is the acceleration due to gravity at the position of a satellite located 300 km above Earth?* |
| Solution | Ratio of acceleration due to gravity at two locations: sea level  and 300 km up |
| 18. | *Verify that the linear speed of an ultracentrifuge is about 0.50 km/s, and Earth in its orbit is about 30 km/s by calculating: (a) The linear speed of a point on an ultracentrifuge 0.100 m from its center, rotating at 50,000 rev/min.(b) The linear speed of Earth in its orbit about the Sun (use data from the text on the radius of Earth’s orbit and approximate it as being circular).* |
| Solution | (a)  (b) |
| 19. | *A rotating space station is said to create “artificial gravity”—a loosely-defined term used for an acceleration that would be crudely similar to gravity. The outer wall of the rotating space station would become a floor for the astronauts, and centripetal acceleration supplied by the floor would allow astronauts to exercise and maintain muscle and bone strength more naturally than in non-rotating space environments. If the space station is 200 m in diameter, what angular velocity would produce an “artificial gravity” of*  *at the rim?* |
| Solution |  |
| 20. | *At takeoff, a commercial jet has a 60.0 m/s speed. Its tires have a diameter of 0.850 m. (a) At how many rev/min are the tires rotating? (b) What is the centripetal acceleration at the edge of the tire? (c) With what force must a determined  kg bacterium cling to the rim? (d) Take the ratio of this force to the bacterium’s weight.* |
| Solution | (a)  (b)  (c)  (d) |
| 21. | ***Integrated Concepts*** *Riders in an amusement park ride shaped like a Viking ship hung from a large pivot are rotated back and forth like a rigid pendulum. Sometime near the middle of the ride, the ship is momentarily motionless at the top of its circular arc. The ship then swings down under the influence of gravity. (a) Assuming negligible friction, find the speed of the riders at the bottom of its arc, given the system’s center of mass travels in an arc having a radius of 14.0 m and the riders are near the center of mass. (b) What is the centripetal acceleration at the bottom of the arc? (c) Draw a free body diagram of the forces acting on a rider at the bottom of the arc. (d) Find the force exerted by the ride on a 60.0 kg rider and compare it to her weight. (e) Discuss whether the answer seems reasonable.* |
| Solution | Assume the top of the circular arc is when the pendulum arm is horizontal.  (a)  (b)  (c)  (d)  The normal force (upward) is three times her weight.  (e) This answer seems reasonable, since she feels like she is being forced into the chair MUCH stronger than just by gravity. |
| 22. | ***Unreasonable Results*** *A mother pushes her child on a swing so that his speed is 9.00 m/s at the lowest point of his path. The swing is suspended 2.00 m above the child’s center of mass. (a) What is the centripetal acceleration of the child at the low point? (b) What force does the child exert on the seat if his mass is 18.0 kg? (c) What is unreasonable about these results? (d) Which premises are unreasonable or inconsistent?* |
| Solution | (a)  (b)  (c) The force in part (b) is very large. The acceleration in part (a) is too much, about 4*g*.  (d) The speed of the swing is too large. At the given speed at the bottom of the swing, there is enough kinetic energy to send the child all the way over the top, ignoring friction. To see this, consider the process of converting the swing’s kinetic energy to potential energy:  Thus, at the given speed at the bottom of the swing, there is enough kinetic energy to send the child all the way over the top, ignoring friction. |

# 6.3 Centripetal Force

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| 23. | *(a) A 22.0 kg child is riding a playground merry-go-round that is rotating at 40.0 rev/min. What centripetal force must she exert to stay on if she is 1.25 m from its center? (b) What centripetal force does she need to stay on an amusement park merry-go-round that rotates at 3.00 rev/min if she is 8.00 m from its center? (c) Compare each force with her weight.* |
| Solution | (a)  (b)  (c) |
| 24. | *Calculate the centripetal force on the end of a 100 m (radius) wind turbine blade that is rotating at 0.5 rev/s. Assume the mass is 4 kg.* |
| Solution |  |
| 25. | *What is the ideal banking angle for a gentle turn of 1.20 km radius on a highway with a 105 km/h speed limit (about 65 mi/h), assuming everyone travels at the limit?* |
| Solution |  |
| 26. | *What is the ideal speed to take a 100 m radius curve banked at a 20.0° angle?* |
| Solution | Using  gives: |
| 27. | *(a) What is the radius of a bobsled turn banked at 75.0° and taken at 30.0 m/s, assuming it is ideally banked? (b) Calculate the centripetal acceleration. (c) Does this acceleration seem large to you?* |
| Solution | (a) For an ideally banked curve:    (b)  (c)  This does not seem too large, but it is clear that bobsledders feel a lot of force on them going through sharply banked turns! |
| 28. | *Part of riding a bicycle involves leaning at the correct angle when making a turn, as seen in Figure 6.36. To be stable, the force exerted by the ground must be on a line going through the center of gravity. The force on the bicycle wheel can be resolved into two perpendicular components—friction parallel to the road (this must supply the centripetal force), and the vertical normal force (which must equal the system’s weight). (a) Show that  (as defined in the figure) is related to the speed  and radius of curvature  of the turn in the same way as for an ideally banked roadway—that is,  (b) Calculate  for a 12.0 m/s turn of radius 30.0 m (as in a race).* |
| Solution | (a)  (b) |
| 29. | *A large centrifuge, like the one shown in Figure 6.37(a), is used to expose aspiring astronauts to accelerations similar to those experienced in rocket launches and atmospheric reentries. (a) At what angular velocity is the centripetal acceleration  if the rider is 15.0 m from the center of rotation? (b) The rider’s cage hangs on a pivot at the end of the arm, allowing it to swing outward during rotation as shown in Figure 6.37(b). At what angle  below the horizontal will the cage hang when the centripetal acceleration is ? (Hint: The arm supplies centripetal force and supports the weight of the cage. Draw a free body diagram of the forces to see what the angle  should be.)* |
| Solution | (a)  (b) |
| 30. | *If a car takes a banked curve at less than the ideal speed, friction is needed to keep it from sliding toward the inside of the curve (a real problem on icy mountain roads). (a) Calculate the ideal speed to take a 100 m radius curve banked at 15.0°. (b) What is the minimum coefficient of friction needed for a frightened driver to take the same curve at 20.0 km/h?* |
| Solution | (a) For an ideally banked curve, we have    (b) |
| 31. | ***Integrated Concepts*** *Modern roller coasters have vertical loops like the one shown in Figure 6.38. The radius of curvature is smaller at the top than on the sides so that the downward centripetal acceleration at the top will be greater than the acceleration due to gravity, keeping the passengers pressed firmly into their seats. (a) What is the speed of the roller coaster at the top of the loop if the radius of curvature there is 15.0 m and the downward acceleration of the car is 1.50g? (b) How high above the top of the loop must the roller coaster start from rest, assuming negligible friction? (c) If it actually starts 5.00 m higher than your answer to the previous part, how much energy did it lose to friction? Its mass is 1500 kg.* |
| Solution | (a)  (b)    (c) |
| 32. | ***Unreasonable Results*** *(a) Calculate the minimum coefficient of friction needed for a car to negotiate an unbanked 50.0 m radius curve at 30.0 m/s. (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?* |
| Solution | (a)  (b) A coefficient of friction this much greater than 1 is unreasonable. For example, the value for rubber on dry concrete is 1.0.  (c) It is unreasonable to go around an unbanked, tight curve so fast. |

# 6.5 Newton’s Universal Law of Gravitation

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| 33. | *(a) Calculate Earth’s mass given the acceleration due to gravity at the North Pole is and the radius of the Earth at the pole is 6371 km. (b) Compare this with the accepted value of .* |
| Solution | (a)  (b) This is identical to the best value to three significant figures. |
| 34. | *(a) Calculate the acceleration due to gravity at Earth due to the Moon. (b) Calculate the acceleration due to gravity at Earth due to the Sun. (c) Take the ratio of the Moon’s acceleration to the Sun’s and comment on why the tides are predominantly due to the Moon in spite of this number.* |
| Solution | (a)  (b)  (c)  The moon has greater effect on the sides since it is the difference in the pull of the body on the near side of the earth vs. the far side. The earth’s diameter is much more significant at lunar distances than at solar distances. |
| 35. | *(a) What is the acceleration due to gravity on the surface of the Moon? (b) On the surface of Mars? The mass of Mars is*  *and its radius is* *.* |
| Solution | (a)  (b) |
| 36. | *(a) Calculate the acceleration due to gravity on the surface of the Sun. (b) By what factor would your weight increase if you could stand on the Sun? (Never mind that you cannot.)* |
| Solution | (a)  (b) |
| 37. | *The Moon and Earth rotate about their common center of mass, which is located about 4700 km from the center of Earth. (This is 1690 km below the surface.) (a) Calculate the acceleration due to the Moon’s gravity at that point. (b) Calculate the centripetal acceleration of the center of Earth as it rotates about that point once each lunar month (about 27.3 d) and compare it with the acceleration found in part (a). Comment on whether or not they are equal and why they should or should not be.* |
| Solution | (a)    (b)  The values are nearly identical. One would expect the gravitational force to be the same as the centripetal force at the core of the system. |
| 38. | *Solve part (b) of Example 6.6 using .* |
| Solution |  |
| 39. | *Astrology, that unlikely and vague pseudoscience, makes much of the position of the planets at the moment of one’s birth. The only known force a planet exerts on Earth is gravitational. (a) Calculate the gravitational force exerted on a 4.20 kg baby by a 100 kg father 0.200 m away at birth (he is assisting, so he is close to the child). (b) Calculate the force on the baby due to Jupiter if it is at its closest distance to Earth, some*  *away.* *How does the force of Jupiter on the baby compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces. (Of course, there could be an unknown force acting, but scientists first need to be convinced that there is even an effect, much less that an unknown force causes it.)* |
| Solution | (a)  (b) The mass of Jupiter is: |
| 40. | *The existence of the dwarf planet Pluto was proposed based on irregularities in Neptune’s orbit. Pluto was subsequently discovered near its predicted position. But it now appears that the discovery was fortuitous, because Pluto is small and the irregularities in Neptune’s orbit were not well known. To illustrate that Pluto has a minor effect on the orbit of Neptune compared with the closest planet to Neptune: (a) Calculate the acceleration due to gravity at Neptune due to Pluto when they are*  *apart, as they are at present. The mass of Pluto is* *. (b) Calculate the acceleration due to gravity at Neptune due to Uranus, presently about*  *apart, and compare it with that due to Pluto. The mass of Uranus is* *.* |
| Solution | (a)  (b)  The effect of Uranus on Neptune’s gravity is 20,000 times stronger than the effect of Pluto. |
| 41. | *(a) The Sun orbits the Milky Way galaxy once each, with a roughly circular orbit averaging light years in radius. (A light year is the distance traveled by light in 1 y.) Calculate the centripetal acceleration of the Sun in its galactic orbit. Does your result support the contention that a nearly inertial frame of reference can be located at the Sun? (b) Calculate the average speed of the Sun in its galactic orbit. Does the answer surprise you?* |
| Solution | (a)  Yes, the centripetal acceleration is so small it supports the contention that a nearly inertial frame of reference can be located at the Sun.  (b) |
| 42. | ***Unreasonable Results*** *A mountain 10.0 km from a person exerts a gravitational force on him equal to 2.00% of his weight. (a) Calculate the mass of the mountain. (b) Compare the mountain’s mass with that of Earth. (c) What is unreasonable about these results? (d) Which premises are unreasonable or inconsistent? (Note that accurate gravitational measurements can easily detect the effect of nearby mountains and variations in local geology.)* |
| Solution | (a)  (b)  (c) That is an enormous mass for one mountain, and its fraction of the earth’s mass is too large.  (d) The unreasonable premise is that the mountain exerts a gravitational force equal to 2.00 % of the person’s weight. |

# 6.6 Satellites and Kepler’s Laws: An Argument for Simplicity

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| 43. | *A geosynchronous Earth satellite is one that has an orbital period of precisely 1 day. Such orbits are useful for communication and weather observation because the satellite remains above the same point on Earth (provided it orbits in the equatorial plane in the same direction as Earth’s rotation). Calculate the radius of such an orbit based on the data for the moon in Table 6.2.* |
| Solution |  |
| 44. | *Calculate the mass of the Sun based on data for Earth’s orbit and compare the value obtained with the Sun’s actual mass.* |
| Solution |  |
| 45. | *Find the mass of Jupiter based on data for the orbit of one of its moons, and compare your result with its actual mass.* |
| Solution | Using:    This result matches the value for Jupiter's mass given by NASA. |
| 46. | *Find the ratio of the mass of Jupiter to that of Earth based on data in Table 6.2.* |
| Solution |  |
| 47. | *Astronomical observations of our Milky Way galaxy indicate that it has a mass of about  solar masses. A star orbiting on the galaxy’s periphery is about  light years from its center. (a) What should the orbital period of that star be? (b) If its period is  instead, what is the mass of the galaxy? Such calculations are used to imply the existence of “dark matter” in the universe and have indicated, for example, the existence of very massive black holes at the centers of some galaxies.* |
| Solution | (a)    (b) |
| 48. | ***Integrated Concepts*** *Space debris left from old satellites and their launchers is becoming a hazard to other satellites. (a) Calculate the speed of a satellite in an orbit 900 km above Earth’s surface. (b) Suppose a loose rivet is in an orbit of the same radius that intersects the satellite’s orbit at an angle of  relative to Earth. What is the velocity of the rivet relative to the satellite just before striking it? (c) Given the rivet is 3.00 mm in size, how long will its collision with the satellite last? (d) If its mass is 0.500 g, what is the average force it exerts on the satellite? (e) How much energy in joules is generated by the collision? (The satellite’s velocity does not change appreciably, because its mass is much greater than the rivet’s.)* |
| Solution | (a) Use  substituting using  and    (b)  In the satellite’s frame of reference, the rivet has two perpendicular velocity components equal to *v* from part (a):    (c) Using kinematics:  (d)  (e) The energy is generated from the rivet. In the satellite’s frame of reference,  So, the change in the kinetic energy of the rivet is: |
| 49. | ***Integrated Concepts*** *(a) Based on Kepler’s laws and information on the orbital characteristics of the Moon, calculate the orbital radius for an Earth satellite having a period of 1.00 h. (b) What is unreasonable about this result? (c) What is unreasonable or inconsistent about the premise of a 1.00 h orbit?* |
| Solution | (a)  (b) This radius is unreasonable because it is less than the radius of Earth .  (c) The premise of a one-hour orbit is inconsistent with the known radius of the Earth. |

# Test Prep for Ap® courses

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| 1. | *Jupiter has a mass approximately 300 times greater than Earth’s and a radius about 11 times greater. How will the gravitational acceleration at the surface of Jupiter compare to that at the surface of the Earth?*  (A) Greater  (B) Less  (C) About the same (D) Not enough information |
| Solution | (a) |
| 2. | *Given Newton’s universal law of gravitation (Equation 6.40), under what circumstances is the force due to gravity maximized?* |
| Solution | *G* is a constant, but the other terms in the equation are all controllable. Maximizing the masses of the objects and minimizing the radius between them results in larger forces. |
| 3. | *In the formula , what does G represent?*  (A) the acceleration due to gravity  (B) a gravitational constant that is the same everywhere in the universe  (C) a gravitational constant that is inversely proportional to the radius  (D) the factor by which you multiply the inertial mass to obtain the gravitational mass |
| Solution | (b) |
| 4. | *Saturn’s moon Titan has a radius of 2.58 × 106 m and a measured gravitational field of 1.35 m/s2. What is its mass?* |
| Solution | Titan’s mass is 1.35 × 1023 kg. |
| 5. | *A recently discovered planet has a mass twice as great as Earth’s, and a radius twice as large as Earth’s. What will be the approximate size of its gravitational field?*  (A) 19 m/s2  (B) 4.9 m/s2  (C) 2.5 m/s2  (D) 9.8 m/s2 |
| Solution | (b) |
| 6. | *Earth is 1.5 × 1011 m from the Sun. Mercury is 5.7 × 1010 m from the Sun. How does the gravitational field of the Sun on Mercury (gSM) compare to the gravitational field of the Sun on Earth (gSE)?* |
| Solution | For convenience, represent the strength of the Sun’s gravitational field on Earth as 1.  The gravitational field is inversely proportional to the square of the radius (or the square of the distance from the center of the Sun’s mass).  For Earth,    For Mercury,      The gravitational field of the Sun is 6.9 times as strong on Mercury as it is on Earth. |

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